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Anomalous interface roughening in 3D porous media: experiment and model

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We report the first imbibition experiments in $2+1$ dimensions – using simple materials as the random media and various aqueous suspensions as wetting fluids. We measure the width $w(l,t)$ of the resulting interface and find it to scale with length l as $w(l,\infty)\sim l^{\alpha}$ with $\alpha=0.50\pm0.05$. This value of α is larger than the value of $\alpha=0.40$ found for the KPZ universality class in $2+1$ dimensions. We develop a new imbibition model that describes quantitatively our experiments. For $d=1+1$, the model can be mapped to directed percolation; for $d=2+1$, it corresponds to a new anisotropic surface percolation problem. Our model leads to the exponent $\alpha=0.5\pm0.05$ in excellent agreement with the experiment.

1. Introduction

Recently considerable progress has been made in understanding the dynamics of non-equilibrium interface growth in the context of a variety of models, analytical theories and experiments (for a recent review, see ref. [1]). Many recent investigations have concentrated on the dynamic scaling properties of the rms interface width

$$w(l,t)\equiv\langle[h(x,t)-\langle h(x,t)\rangle]^2\rangle^{1/2}\sim l^{\alpha}f(t/l^{\alpha/\beta})\,. \tag{1}$$

Here $h(x,t)$ is the surface height at time t, the angular brackets denote the average over x, a vector belonging to a $d-1$ hypercube of size l in $d-1$ hyperplane perpendicular to the direction of growth; also, $f(u)\sim u^{\beta}$ for $u\ll1$ and $f(u)\to\text{const}$ for $u\gg1$.

However, until recently, most of the experiments (see ref. [2] and references therein) were made only in $d=1+1$, where the exponents α and β were found to be larger than the exponents of the $(1+1)$-dimensional KPZ equation [3], $\alpha=1/2$ and $\beta=1/3$. Here we address a question whether the same anomalous behavior can be observed in $2+1$ dimensions and provide a theoretical explanation of our experimental finding.

We report preliminary experiments in which water, ink, coffee and other suspensions are absorbed by a 3D paper-roll or a 3D sponge-like material "Oasis"[#1], forming a rough interface between wet and dry regions. We analyze this morphology and measure its roughness exponent α. We extend a model [4–6] of the interface roughening $d=1+1$ to $d=2+1$. Both the model and the experiment produce interfaces with an anomalously large value of $\alpha = 0.5 \pm 0.05$ (the Kim–Kosterlitz prediction [7] is $\alpha = 2/(d+2) = 0.4$).

Here we study only the properties of saturated interface: its width $w(l) = w(l, t=\infty)$ and the height–height correlation function $c(l) = c(l, t=\infty)$, defined by

$$c(l,t) \equiv \langle |h(\boldsymbol{r}+\boldsymbol{x}, t) - h(\boldsymbol{x}, t)|^2 \rangle^{1/2} . \qquad (2)$$

Here $\boldsymbol{r}$ is a two-dimensional vector of length l, perpendicular to the direction of growth and the angular brackets denote an average over $\boldsymbol{x}$. For $l \ll L$ (where L is the system size), $c(l) \sim l^{\alpha}$. Note that $c(l)$ scales similarly to the width $w(l)$.

2. Experiments

To demonstrate the independence on any particular propagating medium, we perform several independent experiments, each implementing a different propagating medium and liquid. The results of all experiments agreed very well with the predictions of the model value for $\alpha \cong 0.5$ and differed very slightly from each other. The most careful experiments were performed with Bingo-brand ink as a propagating fluid. The reasons for using Bingo ink were the following: high viscosity which results in larger effects of inhomogeneities bringing the system closer to the pinning threshold, good coloring and low price. Also high viscosity diminishes possible changes of interface while cutting the sample.

The first propagating medium we used was a highly porous, green spongy-like material commonly used by florists and referred to as "Oasis". A brick of square horizontal section of size $7 \times 7\,\mathrm{cm}^2$ was placed inside a large beaker, where an approximately constant very slow increase (5 cm per hour) of the level of Bingo ink was maintained by a constant flow of ink via a siphon.

[#1] We thank M. Cieplak for suggesting the use of "Oasis" for 3D experiments.

During the whole experiment the average level of ink in the brick was about 2 cm higher than the level of ink in the beaker (due to capillary effects). This allows us to keep the driving force more or less constant in order to reach a saturated stage of the interface with well developed fluctuations. The ink was allowed to propagate upward throughout the brick for about an hour, after which the brick was then longitudinally sliced so as to scan the interfaces between dry and wet regions.

The slices were immediately scanned using an Apple scanner with a gray scale setting of 300 dots per inch (see fig. 1a). After scanning, the images were transformed to black and white by the special image process software which uses different thresholds of grey in order to achieve the best possible identification of dry and wet areas. Then the function $h(x)$ along the slice was obtained by measuring the height of the highest dark pixel in a column.

The second experiment employed a fine-grained paper roll of 15 cm outer diameter, 3.5 cm inner diameter and 6 cm in height, which was placed on a plate and elevated by several small ball bearings. This was done so as to ensure a smooth, symmetrical flow of ink in the longitudinal direction. The ink was added gradually and periodically, thereby maintaining a fairly constant level of ink for about an hour when the ink in the paper reached an average level of 2.5 cm. After that no further increase of the level of ink in the paper was observed. The roll was then cut and the interfaces between dry and wet regions

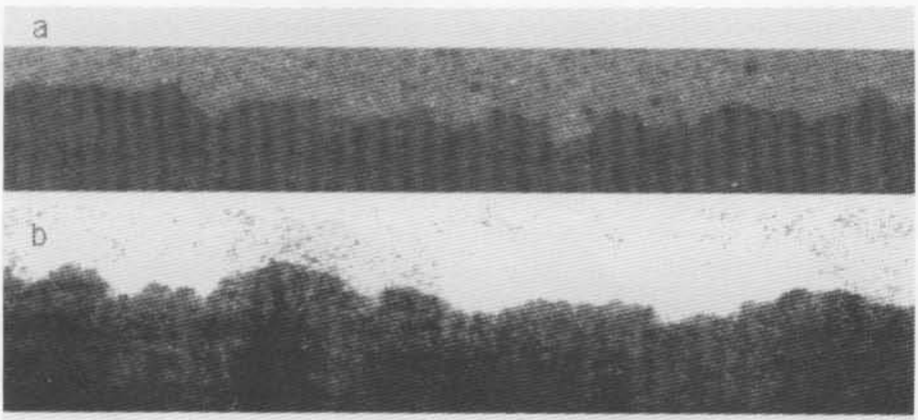

Fig. 1. Digitized ink interface in the "Oasis brick" (a) and paper roll (b), using a Apple Scanner with a resolution of 300 pixels per inch. (c) Half of the actual size of the analyzed paper sheet.

in separate sheets from varying radii were scanned and analyzed the same way as in the previous experiment (see fig. 1b, c).

The average $w(l)$ and $c(l)$ from about 13 slices in the first experiment and 9 sheets in the second experiment were plotted on the double-log plots. All plots are approximately linear over 1.5 decade of l which supports the scaling form $w(l) \sim c(l) \sim l^{\alpha}$ and allows us to get fairly good estimates of α. The values of $\alpha = 0.5 \pm 0.05$ estimated from $w(l)$ and $c(l)$ appear to be the same within the error bar for both experiments. The error bars are determined as a standard deviation of slopes measured for each image separately. The results of the experiments with other liquids are within this quoted error bar. Thus, the value of α we found is definitely above 0.4, the upper boundary for the KPZ universality class.

3. Model

The model we have studied is a straightforward generalization to $d = 2 + 1$ of the $(1 + 1)$-dimensional model studied in refs. [4,5] and can be described as the spreading percolation with anisotropy. The porous medium in our model is simulated by a body-centered cubic lattice with a certain fraction p of blocked cells which represent the inhomogeneities of the media. The horizontal cross-section of the lattice is square of size $L \times L$ with periodic boundary conditions taken on the edges. Every lattice cell can also be "wet" or "dry". At $t = 0$ all lattice cells are dry except for one horizontal layer at the bottom with height $h = 0$. At each time step we take a random point $\boldsymbol{x} = (x_1, x_2)$ in the horizontal cross-section of the lattice and define the highest unblocked dry cell with horizontal coordinates (x_1, x_2) which has at least one nearest-neighboring wet cell. If there is no such unblocked dry cell then no action is taken and we pass to the next time step, otherwise the above defined cell becomes wet together with all cells (blocked, or unblocked does not matter) which are below it in the same column (x_1, x_2). The last rule corresponds to the erosion of overhangs which introduces the anisotropy in the model and makes it different from the regular spreading percolation model. Also this rule implies that the interface between wet and dry regions can be described by a single-valued function $h(x, y)$.

4. Discussion

In $d = 1 + 1$, the wetting process can be stopped by a directed path of blocked cells which occurs in the direction perpendicular to the growth. (See fig. 4 in ref. [5].) The properties of such a path that may occur when the fraction of blocked cells is close to the critical one, $p = p_c = 0.47$, is well known from the theory of directed percolation [8]. It is known that the path of directed percolation is self-affine with roughness exponent $\alpha = 0.63$ [9]. The numerical value for the α interface which is not completely pinned but is still propagating near criticality was found to be slightly larger, $\alpha = 0.7 \pm 0.05$ [5,6], and it is not clear whether the moving interface is self-affine. There is an indication [10] that it could be multi-affine. The results of the $d = 1 + 1$ model are in the excellent agreement with our experiments with propagation of fluid in the single sheet of paper [4,5] or in the thin (effectively two-dimensional) slice of the "Oasis" brick where the roughness exponent of the completely stopped interface was found to be 0.63 ± 0.04.

In $d = 2 + 1$, the wetting in our model can be stopped by a "directed" surface of blocked cells, which does not have overhangs. Unlike in $d = 1 + 1$ there is no direct mapping between this problem and any other known percolation model. However, we can expect from the analogy with $d = 1 + 1$ that this surface is self-affine (see fig. 2). The massive numerical studies of the model for system sizes varying from $L = 32$ to $L = 1024$ and for the fraction of

Fig. 2. An example of the model interface between wet and dry cells for system size 1024×1024 and $p = 0.74$. Almost all the surface is pinned by dry blocked cells. The unblocked dry cells adjacent to the interface are shown in black.

occupied cells close to critical ($p_c = 0.741 \pm 0.001$) shows that this is really the case.

We found that for a completely stopped interface $w(L)$ scales with L like $L^{0.49\pm0.02}$ while there is a large region of l where $c(l) \sim l^{0.46\pm0.02}$ and size of this region as well as the value of the exponent estimate have a tendency to increase with the system size. Thus we can conclude that the true asymptotic value of the exponent α is between 0.46 and 0.52. The data on the moving interface suggest that as in 2D α is slightly larger: $\alpha_{\rm dyn} = 0.52 \pm 0.03$. These results are in the excellent agreement with our experimental results.

We have also generalized the model of Tang and Leschhorn [6] for $d = 3$ and found that $p_c = 0.8$ and that the value of α is the same as for our model. The generalization of both models to $d > 3$ is straightforward and may have significant theoretical interest. The dynamical properties of both models have been reported [2]. The dynamics of 3D fluid flow is also under investigation and further details will be reported elsewhere [11]. Our model can be applied to describe surface erosion may be relevant also to geology. The roughness exponent of certain mountain regions [12] was found to be 0.57 and may represent a crossover between $d = 2$ and $d = 3$ behavior.

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Note added in proof

Recently, S. Havlin and S. Redner (unpublished) have argued that the upper critical dimension for the anisotropic surface model is 3.5, thus supporting the values of $\alpha = \nu_\perp / \nu_\parallel \approx \alpha_{\rm MF} = 1/2$ reported in this work for both experiments and numerical simulations.

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